

THEOREMS ON POLYNOMIAL FUNCTIONS

Fundamental Theorem of Algebra

Every polynomial function of positive degree has **at least one** complex zero.

Theorem on the Number of Zeros of a Polynomial Function

Every polynomial function of degree n has exactly n zeros (complex or real), where a root of multiplicity m is counted m times.

Descartes' Rule of Signs (DRS)

Given a polynomial function $f(x)$ with a non-zero constant term,

- the number of **positive real zeros** of $f(x)$ is equal to the number of variations of sign of $f(x)$ or less than that number by an even integer.
- the number of **negative real zeros** of $f(x)$ is equal to the number of variations of sign of $f(-x)$ or less than that number by an even integer.

Remainder Theorem

If a polynomial function $f(x)$ is divided by $x-c$, then the remainder is $f(c)$.

Factor Theorem

A polynomial function $f(x)$ has a factor $x-c$ if and only if $f(c) = 0$.

Rational Root Theorem (RRT)

If p/q is a rational number in the lowest terms and is a zero of $f(x)$, then p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n .

Theorem on the Upper Bound (UB) and Lower Bound (LB) of Zeros of a Polynomial Function

Suppose that $f(x)$ is a polynomial with a positive leading coefficient and that $f(x)$ is divided by $x - c$ through synthetic division.

- If $c > 0$ and if all numbers in the quotient and remainder are either positive or zero, then c is an upper bound for the real zeros of $f(x)$.
- If $c < 0$ and if the numbers in the quotient and remainder are alternately positive and negative (and a 0 is considered to be either positive or negative), then c is a lower bound for the real zeros of $f(x)$.

Complex Conjugate Roots Theorem

If $a + bi$ (where $a, b \in \mathbb{R}$, $b \neq 0$) is a zero of the polynomial function $f(x)$, then $a - bi$ (its conjugate) is also a zero of $f(x)$.

Square Root Conjugate Theorem

If $a + b\sqrt{c}$ (where $a, b, c \in \mathbb{R}$, $b \neq 0$, c is not a perfect square) is a zero of the polynomial function $f(x)$ with rational coefficients, then $a - b\sqrt{c}$ is also a zero of $f(x)$.