THEOREMS ON POLYNOMIAL FUNCTIONS

Fundamental Theorem of Algebra

Every polynomial function of positive degree has **at least one** complex zero.

Theorem on the Number of Zeros of a Polynomial Function

Every polynomial function of degree **n** has exactly **n** zeros (complex or real), where a root of multiplicity **m** is counted **m** times.

Descartes' Rule of Signs (DRS)

Given a polynomial function f(x) with a non-zero constant term,

a) the number of **positive real zeros** of f(x) is equal to the number of variations of sign of **f(x)** or less than that number by an even integer.
b) the number of **negative real zeros** of f(x) is equal to the number of variations of sign of **f(-x)** or less than that number by an even integer.

Remainder Theorem

If a polynomial function f(x) is divided by **x–c**, then the remainder is **f(c).**

Factor Theorem

A polynomial function f(x) has a factor **x–c** if and only if **f(c) = 0.**

Rational Root Theorem (RRT)

If **p/q** is a rational number in the lowest terms and is a zero of f(x), then **p** is a factor of the constant term a_0 and **q** is a factor of the leading coefficient a_n.

Theorem on the Upper Bound (UB) and Lower Bound (LB) of Zeros of a Polynomial Function

Suppose that f(x) is a polynomial with a positive leading coefficient and that f(x) is divided by x – c through synthetic division.

a) If $c > 0$ and if all numbers in the quotient and remainder are either positive or zero, then **c** is an upper bound for the real zeros of f(x).
b) If $c < 0$ and if the numbers in the quotient and remainder are alternately positive and negative (and a 0 is considered to be either positive or negative), then **c** is a lower bound for the real zeros of f(x).

Complex Conjugate Roots Theorem

If $\mathbf{a + bi}$ (where $a, b \in \Re, b \neq 0$) is a zero of the polynomial function f(x), then $\mathbf{a - bi}$ (its conjugate) is also a zero of f(x).

Square Root Conjugate Theorem

If $\mathbf{a + b\sqrt{c}}$ (where $a, b, c \in \Re, b \neq 0$, c is not a perfect square) is a zero of the polynomial function f(x) with rational coefficients, then $\mathbf{a - b\sqrt{c}}$ is also a zero of f(x).